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ABSTRACT

In most behavioral science research very little attention is ever given to the probability of committing a Type II error, i.e., the probability of failing to reject a false null hypothesis. Recent publications by Cohen have led to insight on this topic for the fixed-effects analysis of variance and covariance. This paper provides social scientists with some insight in dealing with Type II error, and optimum sample size and number of levels in the random-effects analysis of variance. (Author)

OPTIMUM SAMPLE SIZE AND NUMBER OF LEVELS IN THE RANDOM-EFFECTS ANALYSIS OF VARIANCE[1]

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ABSTRACT

In most behavioral science research very little attention is ever given to the probability of committing a Type II error, *i.e.*, the probability of failing to reject a false null hypothesis. Recent publications by Cohen (1970, 1969) have led to a great deal of insight on this topic for the fixed-effects analysis of variance and covariance. It is the purpose of this presentation to provide social scientists with some insight in dealing with Type II error, and, therefore, optimum sample size and number of levels, in the random-effects analysis of variance.

Introduction

The random-effects analysis of variance is described in most statistics texts (Winer, 1972; Glass & Stanley, 1970; Dixon & Massey, 1969; Kirk, 1968; Hays, 1965; Guenther, 1964) used by social scientists. Each of these texts provides the reader with a procedure for selecting sample size for the fixed-effects model. However, none of them provide the reader with guide lines for selecting the number of levels (treatments) or elements within a level (subjects) for random-effects designs. Instead they provide examples in which it appears as though sample size and number of levels were chosen because they "looked good". It is exactly this kind of "looks good" procedure which these same authors have tried to avoid by discussing sample size selection for the fixed-effects model.

Although the random-effects model is not used as often by social scientists as is the fixed-effects model, it certainly does have a wide variety of applications. In this model a researcher is faced with the problem of having to draw inferences about an entire set of distinct treatments or factor levels. In this case the researcher is not interested in the values of the individual treatment effects, as in fixed-effects designs, but in the variance of the population from which these effects were randomly selected. Possible populations which a social scientist might consider for study would be schools, teachers, psychologists, sociologists, time periods (e.g., Glass & Stanley, 1970, pp. 452-462), animal strains, or classes.

For example, suppose that a population of school teachers is available to teach reading to first graders using a certain method.

It is decided that the method will be adopted for use provided its success is not heavily dependent on the personalities of individual teachers. A random sample of teachers is drawn and randomly assigned to a random sample of first grade pupils. The dependent variable (a standardized reading test) is selected and a one-way analysis of variance, random-effects model, is to be used to test the hypothesis that there are no significant differences among teachers in the teacher population.

Researchers faced with the preceding problem are often ignorant of how many teachers to select and the number of students to assign to each teacher. The following discussion provides an answer to these questions for the one-way random-effects analysis of variance, and indicates the problems involved in answering these questions for more complex random-effects designs.

Background

Cohen (1970, 1969) discusses the relationship between power (the probability of rejecting the null hypothesis (H_0) when it is indeed false), sample size, and effect size (a measure of the effects one desires to detect) for the fixed-effects model. The calculation of power in that model is difficult because when H_0 is false the F statistic is no longer distributed as a central F but as a noncentral F. In the random-effects model this is not the case. Here power calculations can be made using the central F distribution.

Hypotheses

In random effects designs, the null hypothesis is generally written

$$H_o\colon\ \theta \leq \theta_o$$

with the alternative being

$$H_A\colon\ \theta > \theta_o.$$

Here $\theta_o \geq 0$ is a preassigned constant, and θ is Cohen's (1969) "effect size" for random effects designs. That is, θ is an index of the degree of departure from the null hypothesis which we want to detect.

Parameter Selection

The major problem faced by investigators is that of choosing values for θ and θ_o. These values may be based on the theory in the area which is being investigated. However, most often, they are informed hunches based on work done in a pilot study or on research found in the literature.

Most beginning statistics texts discuss the rather limited case where $\theta_o = 0$. This leads to the null hypothesis

$$H_o: \theta = 0.$$

This hypothesis may be unrealistic since some differences most likely do exist among levels in the random-effects design. Therefore, values of θ_o of 0.10, 0.50 and 1.00 along with 0.00 were considered in this paper.

It is the writer's firm belief that if a researcher is unable to select values of θ and θ_o then he should consider his experiment nothing more than a pilot study. Not knowing what θ and θ_o probably are prevents one from being able to select an appropriate number of observations with which to test H_o. Using too small of a sample, the researcher may decide to fail to reject a false null hypothesis, Type II error. Using too large of a sample, he may decide to reject a true null hypothesis, Type I error. In either case, he does not know enough about the area in which he is working to select an appropriate sample size. Unfortunately, if such errors occur they may find their way into print and lend confusion to an area of research.

The One-Way Design

In the one-way random-effects design, the null hypothesis is generally written

$$H_o: \sigma_A^2 \leq \theta_0 \sigma_e^2.$$

with the alternative being

$$H_A: \sigma_A^2 > \theta_0 \sigma_e^2.$$

Here σ_A^2 is the variance of the effects for factor A; σ_e^2 is the variance of the sample elements, and $\theta_0 \geq 0$ is a preassigned constant.

Then the power of the F test (Scheffe, 1959) is a function of $\theta = \sigma_A^2/\sigma_e^2$. That is, the ratio $\sigma e^2/\sigma e^2$ is an index of the degree of departure from the null hypothesis which we want to detect. So that,

$$\text{Power} = \Pr \{F(L-1,N-L) \geq F(\alpha;\ L-1,\ N-L)(1+n\theta_o)/(1+n\theta)$$

where

F = the F statistic,

L = the number of levels,

α = the level of significance,

N = the total number of elements,

$n = N/L$, the number of elements in a level of the design.

The above formula for power is adapted from formula (7.2.12) in Scheffe, 1959, p. 227.

That is, power for any one-way random-effects analysis of variance is found using the central F distribution and is calculated by finding the probability of drawing an F value from a central F distribution, with L-1 and N-L degrees of freedom, that is greater than or equal to the value $F(\alpha;L-1,N-L)(1+n\theta_o)/(1+n\theta)$.

Therefore, given θ and θ_o, one may select the optimum number of levels (L) to be used with a fixed total number of people (N) such that the power of the statistical F test is maximized. Here, N/L = n is the number of elements in each level of the design. Equal n's in each level have been assumed since some difficulties arise in the random-effects analysis unless there are equal numbers of observations (Hays, 1965, p. 419).

Tables 1, 2, and 3 were constructed[1] using the preceding procedure with the Central F distribution, in a computer program described in Barcikowski (1972, see Appendix A). In these tables the optimum number of levels for the one-way random-effects model were found for given values of N, α, θ, and θ_o. The procedure used was to select α, θ, θ_o and the total number of observations and then find the number of levels which would yield the largest power for these values. In each case the possible number of levels ranged from two to N/2 since at L = 1, σ_A^2 can not be estimated, and at L = N neither σ_e^2 nor σ_A^2 can be estimated.

For example, in constructing Table 2 where $\theta_o = 0.00$, $\theta = 0.60$, and N = 20 the following values were generated for the levels from two through ten (N/2):

Levels (L)	Power
2	.43750
3	.46739
4	.50688
5	.48910

[1]The author is indebted to Miss Ruth A. Wagenhofer for her effort in compiling the tables.

Table 1

Power Values and Optimum Number

of Levels for Rixed Total Numbers

of Observations in the One-Way

Random-Effects Analysis of Variance

$\alpha = .01$

$\theta_0 = 0.00$

θ/N	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0
10	2,.045	2,.089	2,.132	2,.172	2,.208	2,.341	2,.426	2,.485
20	2,.114	2,.214	2,.214	4,.394	4,.471	5,.706	5,.706	5,.881
30	3,.180	3,.348	3,.348	5,.586	5,.668	6,.875	6,.875	10,.977
40	3,.246	4,.463	4,.463	5,.716	8,.795	10,.954	10,.954	13,.995
50	3,.306	5,.561	5,.561	7,.809	10,.878	10,.981	10,.981	16,.999
60	4,.383	6,.644	6,.644	10,.881	10,.930	15,.994	15,.994	12,1.00
80	5,.439	7,.713	7,.713	10,.924	14,.960	14,.998	14,.998	10,1.00
80	5,.497	8,.770	8,.770	10,.950	16,.977	20,.999	20,.999	9,1.00
90	5,.547	9,.817	9,.817	15,.970	18,.988	15,1.00	15,1.00	8.1.00
100	5,.591	10,.855	10,.855	20,.980	20,.993	14,1.00	14,1.00	8,1.00
300	15,.965	30,.999	30,.999	7,1.00	6,1.00	6,1.00	6,1.00	5,1.00
500	15,1.00	8,1.00	8.1.00	7,1.00	6,1.00	6,1.00	6,1.00	5,1.00

$\theta_0 = 0.10$

θ/N	0.3	0.5	0.7	0.9	1.0	2.0	3.0	4.0
10	2,.032	2,.059	2,.089	2,.118	2,.132	2,.250	2,.334	2,.396
20	2,.057	4,.120	4,.194	4,.267	4,.302	5,.567	5,.718	5,.804
30	3,.082	5,.188	5,.302	6,.405	6,.454	6,.751	10,.891	10,.948
40	4,.106	5,.252	8,.395	8,.527	8,.582	10,.878	10,.958	13,.984
50	5,.130	7,.309	10,.484	10,.629	10,.686	10.,931	16,.984	16,.996
60	6,.153	10,.371	10,.569	12,.613	12,.768	15,.971	20,.996	20,.999
70	7,.176	10,.430	14,.636	14,.781	14,.830	23,.985	23,.999	17,1.00
80	8,.199	10,.479	16,.697	16,.832	16,.877	20,.994	26,1.00	15,1.00
90	10,.223	15,.530	15,.751	18,.876	18,.912	30,.997	18,1.00	14,1.00
100	10,.245	14,.569	20,.795	20,.907	20,.978	25,.999	16,1.00	12,1.00
300	30,.632	50,.967	60,.998	50,1.00	30,1.00	17,1.00	10,1.00	8,1.00
500	62,.850	50,.998	22,1.00	16,1.00	15,1.00	10,1.00	8,1.00	8,1.00

Table 1 continued

$\theta_o = 0.50$

N \ θ	0.7	0.8	0.9	1.0	2.0	3.0	4.0	5.0
10	2,.018	2,.023	2,.028	2,.034	2,.095	2,.155	2,.208	3,.259
20	4,.024	4,.034	4,.045	4,.057	5,.218	5,.380	5,.508	5,.604
30	5,.029	6,.043	6,.060	6,.080	10,.329	10,.570	10,.730	10,.828
40	8,.034	8,.053	8,.076	8,.102	10,.445	13,.344	13,.847	13,.919
50	10,.039	10,.062	10,.091	10,.125	16,.526	16,.799	16,.916	16,.963
60	12,.043	12,.071	12,.107	15,.149	20,.634	20,.885	20,.964	20,.988
70	14,.048	14,.081	14,.122	14,.171	23,.703	23,.926	23,.982	23,.995
80	16,.052	16,.090	20,.139	20,.197	20,.762	26,.953	26,.991	40,.998
90	18,.056	18,.099	18,.154	18,.218	30,.823	30,.975	30,.996	45,.999
100	20,.061	20,.109	25,.175	25,.246	33,.856	33,.985	33,.998	30,1.00
500	125,.255	125,.511	125,.739	125,.884	68,1.00	20,1.00	15,1.00	13,1.00
1000	250,.503	250,.828	333,.966	190,1.00	35,1.00	20,1.00	15,1.00	13,1.00

$\theta_o = 1.00$

N \ θ	1.2	1.4	1.6	1.8	2.0	3.0	4.0	5.0
10	2,.015	2,.020	2,.025	2,.032	2,.038	2,.074	2,.111	2,.146
20	4,.017	5,.027	5,.039	5,.054	5,.070	5,.166	5,.270	5,.365
30	6,.020	6,.034	6,.052	6,.073	10,.098	10,.260	10,.429	10,.570
40	10,.022	10,.040	10,.065	10,.096	10,.132	10,.344	13,.550	13,.702
50	10,.024	10,.046	10,.075	12,.113	12,.157	16,.425	16,.653	16,.799
60	15,.026	15,.053	15,.091	15,.140	20,.198	20,.525	20,.760	20,.885
70	14,.027	17,.058	23,.102	23,.161	23,.229	23,.592	23,.821	23,.926
80	20,.029	20,.065	20,.118	20,.185	20,.262	26,.651	26,.869	40,.955
90	22,.031	30,.071	30,.132	30,.211	30,.302	30,.721	30,.914	30,.975
100	25,.033	25,.078	25,.145	33,.233	33,.333	33,.765	33,.938	50,.985
500	166,.103	166,.365	166,.678	166,.882	166,.967	94,1.00	34,1.00	24,1.00
1000	333,.202	333,.677	333,.944	250,1.00	142,1.00	52,1.00	34,1.00	24,1.00

Table 2

Power Values and Optimum Number

of Levels for Fixed Total Numbers

of Observations in the One-Way

Random-Effects Analysis of Variance

$\alpha = .05$

$\theta_0 = 0.00$

	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0
10	2,.142	2,.220	2,.262	2,.333	2,.374	3,.518	3,.622	5,.693
20	2,.241	4,.386	4,.507	4,.596	4,.662	5,.847	5,.914	5,.945
30	3,.342	5,.530	5,.666	6,.756	6,.818	10,.949	10,.984	10,.994
40	4,.424	5,.645	8,.768	8,.854	8,.904	10,.984	13,.996	13,.999
50	5,.495	5,.724	10,.843	10,.914	10,.950	16,.994	16,.999	12,1.00
60	5,.564	6,.792	10,.900	12,.950	12,.974	20,.999	12,1.00	10,1.00
70	5,.621	7,.843	10,.934	14,.171	14,.987	23,1.00	10,1.00	8,1.00
80	5,.668	10,.883	10,.955	16,.983	16,.993	13,1.00	9,1.00	8,1.00
90	6,.714	10,.913	15,.965	18,.997	18,.997	11,1.00	8,1.00	7,1.00
100	7,.746	10,.933	14,.980	20,.995	20,.998	10,1.00	8,1.00	7,1.00
300	20,.989	23,1.00	8,1.00	7,1.00	6,1.00	5,1.00	5,1.00	5,1.00
500	10,1.00	7,1.00	7,1.00	6,1.00	6,1.00	5,1.00	5,1.00	5,1.00

$\theta_0 = 0.10$

	0.3	0.5	0.7	0.9	1.0	2.0	3.0	4.0
10	2,.112	2,.169	2,.220	2,.263	2,.282	3,.436	3,.547	5,.628
20	4,.163	5,.283	4,.386	5,.473	5,.512	5,.753	5,.854	10,.907
30	5,.211	5,.377	6,.513	6,.618	6,.661	1-,.884	10,.962	10,.984
40	5,.255	8,.445	8,.616	8,.727	10,.771	10,.952	13,.988	13,.996
50	7,.290	10,.526	10,.698	10,.806	10,.843	16,.978	16,.996	16,.999
60	10,.325	10,.592	12,.764	15,.863	15,.897	20,.993	20,.999	14,1.00
70	10,.364	10,.644	14,.816	14,.904	14,.930	23,.997	17,1.00	13,1.00
80	10,.397	16,.692	16,.857	20,.934	20,.955	26,.999	15,1.00	11,1.00
90	10,.426	15,.638	18,.890	18,.954	18,.969	30,1.00	14,1.00	10,1.00
100	11,.453	20,.771	20,.915	25,.969	25,.981	20,1.00	12,1.00	10,1.00
300	37,.821	50,.991	48,1.00	26,1.00	21,1.00	15,1.00	8,1.00	7,1.00
500	71,.949	29,1.00	17,1.00	13,1.00	12,1.00	9,1.00	8,1.00	7,1.00

Table 2 continued

$\theta_0 = 0.50$

N \ θ	0.7	0.8	0.9	1.0	2.0	3.0	4.0	5.0
10	2,.076	2,.090	2,.103	2,.116	3,.239	3,.343	5,.429	5,.509
20	5,.095	5,.121	5,.147	5,.175	5,.429	5,.601	10,.720	10,.809
30	6,.109	6,.144	6,.181	6,.218	10,.578	10,.784	10,.884	10,.934
40	8,.122	10,.167	10,.215	10,.265	13,.677	13,.870	20,.945	20,.978
50	10,.134	10,.186	10,.242	10,.300	16,.755	16,.924	25,.977	25,.993
60	15,.145	15,.207	15,.275	15,.344	20,.832	20,.963	20,.991	30,.998
70	14,.155	14,.224	14,.298	17,.373	23,.875	23,.979	35,.996	35,.999
80	20,.166	20,.245	20,.330	20,.415	26,.907	26,.988	40,.999	24,1.00
90	18,.175	18,.260	22,.351	30,.[illegible]	30,.938	30,.994	45,.999	21,1.00
100	25,.186	25,.281	25,.381	25,.480	33,.955	33,.997	50,1.00	20,1.00
500	125,.499	125,.749	125,.899	166,.967	27,1.00	16,1.00	13,1.00	11,1.00
1000	250,.745	333,.944	237,1.00	115,1.00	27,1.00	16,1.00	13,1.00	11,1.00

$\theta_0 = 1.00$

N \ θ	1.2	1.4	1.6	1.8	2.0	3.0	4.0	5.0
10	2,.065	2,.081	2,.096	3,.112	3,.129	3,.209	3,.281	5,.350
20	5,.076	5,.105	5,.137	5,.169	5,.203	5,.361	10,.491	10,.610
30	10,.083	10,.123	10,.168	10,.216	10,.267	10,.501	10,.672	10,.784
40	10,.090	10,.139	10,.195	10,.255	13,.317	13,.595	13,.771	20,.878
50	12,.094	16,.152	16,.219	16,.291	16,.365	16,.672	25,.848	25,.935
60	15,.101	20,.170	20,.251	20,.338	20,.424	20,.756	20,.906	30,.966
70	23,.106	23,.183	23,.274	23,.371	23,.466	23,.805	35,.938	35,.982
80	20,.112	20,.196	26,.296	26,.402	26,.505	26,.845	40,.961	40,.991
90	30,.116	30,.212	30,.325	30,.442	30,.533	30,.887	45,.976	45,.996
100	25,.121	33,.224	33,.346	33,.471	33,.587	33,.911	50,.985	50,.998
500	166,.277	166,.625	166,.868	166,.967	166,.993	77,1.00	25,1.00	19,1.00
1000	333,.438	333,.867	333,.992	142,1.00	100,1.00	41,1.00	25,1.00	19,1.00

Table 3

Power Values and Optimum Number

of Levels for Fixed Total Numbers

of Observations in the One-Way

Random-Effects Analysis of Variance

$\alpha = .10$

$\theta_o = 0.00$

N \ θ	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0
10	2,.225	2,.314	2,.380	2,.243	2,.470	3,.633	5,.737	5,.808
20	4,.331	4,.501	4,.615	5,.694	5,.755	5,.897	5,.944	10,.972
30	3,.444	5,.637	6,.754	6,.830	6,.877	10,.973	10,.992	10,.997
40	4,.531	5,.734	8,.843	8,.906	10,.941	10,.992	13,.998	13,1.00
50	5,.602	7,.801	10,.900	10,.948	10,.971	16,.997	16,1.00	10,1.00
60	5,.662	10,.855	10,.938	12,.971	15,.986	20,.999	10,1.00	8,1.00
70	7,.711	10,.897	10,.960	14,.984	14,.993	14,1.00	9,1.00	7,1.00
80	8,.753	10,.926	10,.975	16,.991	20,.997	10,1.00	8,1.00	7,1.00
90	6,.791	10,.945	15,.985	18,.995	18,.998	9,1.00	8,1.00	6,1.00
100	9,.821	11,.959	20,.990	20,.998	20,.999	9,1.00	7,1.00	6,1.00
300	20,.994	20,1.00	7,1.00	7,1.00	6,1.00	5,1.00	5,1.00	5,1.00
500	9,1.00	7,1.00	6,1.00	6,1.00	6,1.00	5,1.00	5,1.00	5,1.00

$\theta_o = 0.10$

N \ θ	0.3	0.5	0.7	0.9	1.0	2.0	3.0	4.0
10	2,.188	2,.258	2,.314	3,.360	3,.385	3,.558	5,.678	5,.758
20	4,.258	4,.396	5,.505	5,.592	5,.628	5,.828	10,.907	10,.953
30	5,.316	6,.496	6,.628	6,.720	6,.754	10,.939	10,.980	10,.992
40	5,.363	5,.558	8,.721	10,.816	10,.850	13,.974	13,.994	20,.998
50	7,.406	10,.646	10,.791	10,.873	10,.900	16,.989	16,.998	25,1.00
60	10,.448	12,.702	12,.844	15,.918	15,.940	20,.997	20,1.00	12,1.00
70	10,.487	14,.750	14,.883	14,.943	14,.960	23,.999	14,1.00	10,1.00
80	10,.520	16,.790	16,.913	20,.964	20,.977	26,1.00	13,1.00	10,1.00
90	15,.550	15,.824	18,.935	18,.975	22,.984	18,1.00	11,1.00	9,1.00
100	14,.577	20,.853	20,.952	25,.984	25,.991	16,1.00	11,1.00	9,1.00
300	50,.892	60,.996	33,1.00	23,1.00	21,1.00	10,1.00	8,1.00	7,1.00
500	71,.975	23,1.00	15,1.00	12,1.00	11,1.00	8,1.00	7,1.00	7,1.00

Table 3 continued

$\theta_0 = 0.50$

N \ θ	0.7	0.8	0.9	1.0	2.0	3.0	4.0	5.0
10	2,.140	2,.158	3,.177	3,.196	3,.355	5,.483	5,.582	5,.657
20	5,.169	5,.205	5,.241	5,.276	5,.551	10,.720	10,.829	10,.892
30	6,.189	6,.236	10,.284	10,.333	10,.700	10,.863	15,.933	15,.968
40	10,.208	10,.268	10,.328	10,.386	13,.784	13,.925	20,.974	20,.991
50	10,.223	10,.291	10,.358	12,.423	16,.845	25,.961	25,.990	25,.997
60	15,.240	15,.319	15,.399	15,.474	20,.901	20,.982	30,.997	30,.999
70	14,.253	17,.338	17,.424	23,.507	23,.930	23,.990	35,.999	23,1.00
80	20,.268	20,.365	20,.460	20,.548	26,.950	40,.995	40,1.00	20,1.00
90	18,.279	22,.382	30,.485	30,.580	30,.969	30,.998	30,1.00	18,1.00
100	25,.294	25,.407	25,.515	25,.612	33,.978	50,.999	25,1.00	20,1.00
500	125,.636	125,.845	125,.947	166,.985	23,1.00	14,1.00	12,1.00	10,1.00
1000	250,.840	333,.976	142,1.00	93,1.00	23,1.00	14,1.00	12,1.00	10,1.00

$\theta_0 = 1.00$

N \ θ	1.2	1.4	1.6	1.8	2.0	3.0	4.0	5.0
10	3,.124	3,.149	3,.173	3,.196	3,.219	3,.330	5,.422	5,.502
20	5,.141	5,.184	5,.227	5,.269	5,.310	10,.492	10,.639	10,.743
30	10,.152	10,.211	10,.272	10,.333	10,.392	10,.632	10,.779	15,.871
40	10,.162	10,.232	13,.305	13,.379	13,.449	13,.716	20,.864	20,.936
50	15,.171	16,.253	16,.340	16,.426	16,.506	16,.787	25,.918	25,.969
60	20,.180	20,.275	20,.375	20,.472	20,.561	20,.846	30,.951	30,.985
70	23,.187	23,.292	23,.402	23,.507	23,.602	23,.883	35,.971	35,.993
80	20,.194	26,.308	26,.427	26,.540	26,.639	26,.913	40,.983	40,.997
90	30,.202	30,.328	30,.459	30,.580	30,.683	30,.938	45,.990	45,.999
100	33,.209	33,.343	33,.482	33,.608	33,.613	50,.953	50,.994	50,.999
500	166,.408	166,.759	166,.929	166,.985	166,.997	69,1.00	22,1.00	17,1.00
1000	333,.580	333,.928	233,1.00	116,1.00	82,1.00	36,1.00	22,1.00	17,1.00

Levels (L)	Power
6	.40543
7	.23867
8	.26410
9	.28892
10	.31314

Maximum power (.50688) is reached when there are four levels with five (N/L) elements under each level. The values entered in the table were 4, .507.

Tables 1, 2, and 3 have alpha levels of .01, .05, and .10 respectively. Each of the tables has θ_0 set at 0.00, 1.10, 0.50, and 1.00, with selected values of and N. These values were selected so as to provide information for common one-way random-effects designs.

The power values in the tables have been rounded to three decimal places. In certain situations sample size was so large that power exceeded .9995 for many values of L. When this happened, the first value of L whose corresponding power exceeded .9995 was selected as being the optimum number of levels to use in the analysis of variance. Since optimum L generally increases with increasing values of θ or N the preceding process explains why some of the values of L decreased with increasing θ or N.

One should be aware of the fact that the tables presented here provide "optimal" sample size and number of levels. Given specified parameters, one can not take different values of L and find larger power. However, an experiment having slightly less power might be satisfactory if the number of levels is limited by cost or some other

factor. For example, if $\alpha = .10$, $\theta_0 = 0.10$, $N = 60$ and $\theta = 1.0$, maximum power (.940) is found when $L = 15$ and $n = 4$. Consider the same parameters when less than maximum power is desired, then with L at 10, $n = 6$, power = .928; with $L = 5$, $n = 12$, power = .845. Such power values may be found for situations not covered in the tables presented by using extensive F tables such as those found in Graybill (1961) or by using a computer program described in Barcikowski (1972), the mainline of which is in appendix A.

Example

The following discussion is based on the problem mentioned earlier concerned with teaching first graders how to read. Four different cases will be presented with the availability of teachers (levels) and students (observations) varying in each case.

Suppose that previous research has indicated that teachers under conditions similar to those in this experiment are about one-tenth as variable as their students; i.e., $\theta_0 = 0.10$. Based on the investigator's experience, e.g., a pilot study, he has decided that it would not be worth the effort to use the method of teaching reading if the ratio of the variability among teachers to the variability among students is 1.0 or larger. This procedure leads him to the following hypothesis and its alternative:

$$H_o: \sigma_A^2 \leq .10\sigma_e^2$$
$$H_o: \sigma_A^2 > .10\sigma_e^2$$

That is, the reading method will not be adopted for use if H_o is rejected, but will be adopted if the researcher fails to reject H_o.

Power for this experiment is based on the fact that if $\theta \geq 1.0$ we would like to detect this with high probability. Armed with this information the researcher may now turn to Tables 1, 2, and 3 and select the number of teachers, L. and the total number of students, N, which will keep the probability of making a Type II error small. His selection of an α level may be dependent on the availability of teachers and/or students to achieve a certain power. However, let us assume he has selected $\alpha = .05$, and then consider the following situations which will influence his selection of a power value.

Case I: L and N Plentiful

Optimally the researcher is in a position where he may select any number of students and teachers. If this is the case he would turn to Table 2 under $\theta_0 = .10$, $\theta = 1.0$ and find the power desired for this experiment. If he selected power at .98 then he might use 25 teachers with each teacher teaching 5 pupils. However, if he selected power at 1.00, 12 teachers with 41 pupils 500/12 each might also be selected. Here one would not need to use N = 500 pupils in order to achieve maximum power but only 12 X 41 = 492.

Case II: L and N Fixed

Consider the problem where due to costs there are a fixed number of teachers and pupils. Let the maximum number of teachers and students that can be sampled at random from their respective populations be 15 teachers and [illegible]50 pupils. Then Table 2 indicates that using only 10 teachers w[illegible] 5 pupils each will yield an experiment having maximum power of .843[illegible]ere using the extra teachers will

only result in a statistical test having less power.

If there were a limit of 20 teachers and 75 students then the power of the statistical test using the optimum number of levels would lie between .930 and .955. Here 15 teachers would probably be a good choice since 15 divides into 75 evenly and is in the range of number of levels, 14-20, considered optimal for $\theta = 1.0$. However, one would certainly want to check this by hand calculation.

Case III: L Fixed; N Plentiful

If there are a fixed number of teachers and plenty of students the researcher may simply look in Table 2 at $\theta_0 = 0.10$ under $\theta = 1.0$ until power appeared to be sufficient. If the investigator desired power at .80 and he had 15 teachers then he could use 10 teachers and 5 students per teacher. If he had fewer than 3 teachers and desired power at .80 then he would have to do his calculations by hand.

Case IV: L Plentiful; N Fixed

If the number of students is fixed at 80 then the optimum number of teachers to select would be 20. In this case the power of the experiment would be .955. Here again, the researcher may not be satisfied with the student-teacher ratio, but .955 is the largest power he can achieve using 80 students. If he is willing to have less power, then using hand caculations he will find using 8 teachers with 10 students each results in a power of .906 or 5 teachers with 16 students each results in a power of .821.

The Two-Way Design

In the two-way random-effects design, one may test three possible null hypotheses. They are the hypotheses concerned with the variance of the main effects, factors A and B, and the variance of the interaction effects between factors A and B. The interaction hypothesis would only be tested provided the number of observations in a cell was two or more. In this presentation it is assumed that the number of observations in each cell are equal.

The null hypothesis concerned with factor A would be written

$$H_{OA}: \theta_A \leq \theta_{oA}$$

with the alternative

$$H_{AA}: \theta_A \geq \theta_{oA}.$$

The null hypothesis concerned with factor B would be written

$$H_{OB}: \theta_B \leq \theta_{oB}.$$

with the alternative

$$H_{AB}: \theta_B \geq \theta_{oB}$$

The null hypothesis concerned with the interaction of factors A and B would be written

$$H_{OAB}: \theta_{AB} \leq \theta_{oAB}.$$

Where $\theta_A = \sigma_A^2/(\sigma_e + n\sigma_{AB})$; $\theta_B = \sigma_B^2/(\sigma_e^2+\sigma_{AB}^2)$; $\theta_{AB} = \sigma_{AB}^2/\sigma_e^2$, and θ_{oA}, θ_{oB}, and $\theta_{o_{AB}}$ are preassigned constants, greater than or equal to zero. Here the variables are defined as follows:

σ_A^2 = the variance of the effects for factor A,

σ_B^2 = the variance of the effects for factor B,

σ_{AB}^2 = the variance of the interaction effects,

σ_e^2 = the error variance,

n = the number of observations in a cell of the design.

The power of the F test (Scheffe, 1954) on H_{oA}, H_{oB} and H_{oAB} may be calculated using

$$\text{Power} = \Pr\{F(I-1,V_{AB}) \geq F(\alpha;I-1,V_{AB})(1+Jn\theta_{oA})/(1+Jn\theta_{A})\};$$

$$\text{Power} = \Pr\{F(J-1,V_{AB}) \geq F(\alpha;I-1,V_{AB})(1+In\theta_{oB})/(1+In\theta_{B})\}$$

$$\text{Power} = \Pr\{F(V_{AB},V_{e}) \geq F(\alpha;V_{AB},V_{e})(1+n\theta_{oAB})/(1+n\theta_{AB})\},$$

respectively. Here the parameters are defined as:

F = the F statistic,

α = the level of significance,

I = the number of levels in factor A,

J = the number of levels in factor B,

$V_e = IJ(n-1)$

$V_{AB} = (I-1)(J-1)$

Using the preceding power formulas with the central F distribution a computer program[1] (Appendix B) was written to calculate power for the statistical F tests used to test the three possible null hypotheses. The procedure used was to select α, θ, θ_o and the total number of observations (N) and then find the number of levels which would yield the largest power for these values across the tests of H_{oA}, H_{oB}, and H_{oAB}. In each case the number of levels of B (or A) ranged from 2 to N/4 while the number of levels of A (or B) ranged from N/4 to 2. This procedure was used since if I or J was one, then either σ_A or σ_B can not be estimated, and if I or J was greater than N/4, σ_{AB} can not be estimated.

[1]One minor limitation of this program is that it assumes $\theta_A = \theta_B = \theta_{AB}$ and $\theta_{oA} = \theta_{oB} = \theta_{oAB}$. Given different values of θ and θ_o one can bypass this limitation by submitting the program once for each of the different pairs of θ and θ_o, and then making comparisons of levels across programs.

Example

Tables 4, 5 and 6 provide an example of the optimum number of levels to test θ_A, θ_B and θ_{AB}, respectively. In all three tables $\alpha = .05$, $\theta = .80$, $\theta_\beta = 0.0$ and $N = 40$. The selection of the preceding parameters would prevent the investigator with a problem. In table 4, power is optimal .777, for testing H_{0A} when there are two observations in a cell, $I = 5$ and $J = 4$. In table 5, power is optimal, .777, for testing H_{0B} when there are two observations in a cell, $I = 4$ and $J = 5$. That is, maximum power is achieved for testing H_0 on each of the factors when the number of levels is five. This is not a major problem since in both cases the power for testing H_{0A} or H_{0B} slips to .764. So that, if one tested H_{0B} with $I = 4$ and $J = 5$ power would be .777 for this test and .764 for the test of H_{0A}. In table 6 the test of H_{0AB} achieves maximum power, .638, when there are four observations in a cell and $J = 2$, $I = 5$ or $I = 2$ and $J = 5$. Under these conditions the power of the test of main effects drops to .538 and .555 depending the number of levels selected for factors A and B.

In this example it would seem that one would be forced to increase sample size and then test the interaction hypothesis. If one failed to reject the interaction hypothesis and if power was not sufficient for the main effects test, a second experiment would have to be run with different levels to test the main effects.

Table 4

Power Values for Factor A

K, The Number of Observations in a Cell	J, The Number of Levels in Factor B	I, The Number of Levels in Factor A	Power
2	2	10	0.658
2	3	6	0.720
2	4	5	0.777
2	5	4	0.764
2	6	3	0.689
2	7	2	0.510
2	8	2	0.545
2	9	2	0.573
2	10	2	0.597
3	2	6	0.599
3	3	4	0.649
3	4	3	0.638
3	5	2	0.484
3	6	2	0.541
4	2	5	0.555
4	3	3	0.567
4	4	2	0.455
4	5	2	0.538
5	2	4	0.490
5	3	2	0.355
5	4	2	0.496
6	2	3	0.358
6	3	2	0.387
7	2	2	0.171
8	2	2	0.181
9	2	2	0.191
10	2	2	0.200

Table 5

Power Values for Factor B

K, The Number of Observations in a Cell	J, The Number of Levels in Factor B	I, The Number of Levels in Factor A	Power
2	2	10	0.597
2	3	6	0.689
2	4	5	0.764
2	5	4	0.777
2	6	3	0.720
2	7	2	0.491
2	8	2	0.553
2	9	2	0.608
2	10	2	0.658
3	2	6	0.541
3	3	4	0.638
3	4	3	0.649
3	5	2	0.464
3	6	2	0.559
4	2	5	0.538
4	3	3	0.567
4	4	2	0.429
4	5	2	0.555
5	2	4	0.496
5	[illegible]	2	0.321
5	4	2	0.490
6	2	3	0.387
6	3	2	0.358
7	2	2	0.171
8	2	2	0.181
9	2	2	0.191
10	2	2	0.200

Table 6

Power Values for Interaction of Factors A and B

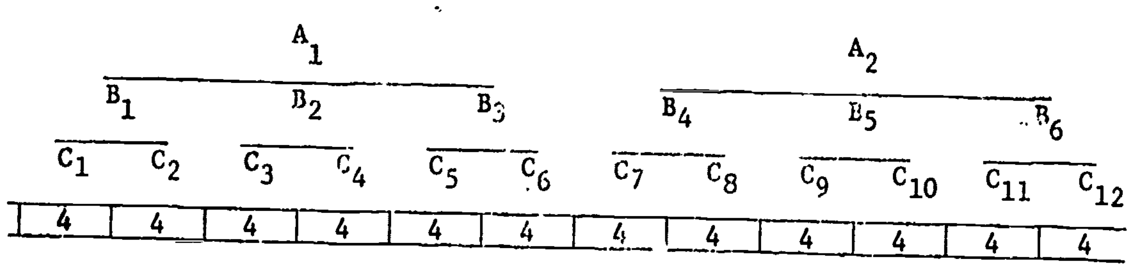
K, The Number of Observations in a Cell	J, The Number of Levels in Factor B	I, The Number of Levels in Factor A	Power
2	2	10	0.528
2	3	6	0.531
2	4	5	0.582
2	5	4	0.582
2	6	3	0.531
2	7	2	0.412
2	8	2	0.453
2	9	2	0.492
2	10	2	0.528
3	2	6	0.580
3	3	4	0.624
3	4	3	0.624
3	5	2	0.514
3	6	2	0.580
4	2	5	0.638
4	3	3	0.632
4	4	2	0.552
4	5	2	0.638
5	2	4	0.632
5	3	2	0.516
5	4	2	0.632
6	2	3	0.571
6	3	2	0.571
7	2	2	0.430
8	2	2	0.458
9	2	2	0.482
10	2	2	0.503

Although in practice one would probably have different values of θ_A, θ_B and θ_{AB}, with different 's, the preceding example provides some indication of the difficulties one might encounter in selecting optimum numbers of levels to arrive at optimum power in the two-way random-effects analysis of variance.

The Nested Design

In considering nested designs these are as many hypotheses to be tested as there are factors. In this design interactions are assumed to be nonexistant. The design discussed here is composed of three factors, A, B and C, from which the reader may generalize to other nested designs. There are an equal number of observations, n, in each cell and the number of levels nested within a level of a factor are equal. There are I levels of A; J levels of B nested within each level of A; K levels of C are nested within each level of B. Figure L provides an example where I = 2, J = 3, K = 12, and n = 4. In figure 1 there are 4 subjects in each cell, 3 levels of B are nested within each level of A and two levels of C are nested within each level of B.

Figure 1
An Example Nested Design



The null hypothesis concerned with each factor would be written

$$H_{ox}: \theta_x \leq \theta_{ox}$$

with the alternative being

$$H_{Ax}: \theta_x \leq \theta_{ox}$$

where x may take on the factor letter designations A, B, or C. Where

$$\theta_A = \frac{\sigma^2{}_A}{\sigma^2{}_e + n\sigma^2{}_e + Kn\sigma^2_B}\,; \qquad \theta_B = \frac{\sigma^2{}_B}{\sigma^2_e + n\sigma^2_e}\,;$$

$$\theta_C = \frac{\sigma^2_e\,\sigma}{\sigma^2_e}\,;$$

θ_{ox} are preassigned constants.

The power of the F test (Scheffe, 1954) on H_{oA}, H_{oB}, and H_{oC} may be calculated using

Power = $\Pr\{F(I-1,V_B) \geq F(\alpha;I-1,V_B)(1+JKn\theta_{oA})/(1+JKn\theta_A)\}$;

Power = $\Pr\{F(V_B,B_C) \geq F(\alpha;V_B,V_C)(1+Kn\theta_{oB})/(1+Kn\theta_B)\}$;

Power = $\Pr\{F(V_e,V_B) \geq F(\alpha;V_e,V_B)(1+n\theta_{oC})/(1+n\theta_C)\}$,

respectively. Here the parameters are defined as:

F = the F statistic,

α = the level of significance,

I = the number of levels in factor A,

J = the number of levels in factor B, nested within each level of A,

K = the number of levels in factor C, nested within each level of B,

$V_B = I(J-1)$, $\quad V_c = JI(K-1)$, $\quad V_e = IJK(n-1)$.

Given a fixed θ_X and θ_{oX}, the same problems encountered in the two-way analyses will be encountered with nested designs. That is, the power for each factor varies and is partially dependent on the number of levels in the other factors. Here the problem is even more complex since there is a large number of designs which can be generated given flexibility in sample size and number of levels for each factor.

Summary

This presentation furnishes researchers in the social sciences with procedures for selecting sample size and optimum number of levels for the random-effects analysis of variance. Hopefully, in so doing, it has brought to the attention of these researchers problems which could be investigated using this analysis of variance model.

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Appendix A[2]

[1]Only the MAINLINE of the program is included here. The subroutines FSTAT and PLEVEL are very long and were excluded. FSTAT returns the F statistic (F) given alpha (ALPHA) and degrees of freedom (D1, D2); PLEVEL returns the probability of a given F statistic (P) given degrees of freedom (D1, D2). Similar routines should be available at most computer centers.

```
C
C
C  PROGRAM 'RANDOM EFFECTS-POWER' WAS PROGRAMMED BY ROBERT S.
C  BARCIKOWSKI AT OHIO UNIVERSITY 1971-1972
C
C  THE PROGRAM IS DESCRIBED IN:  EDUCATIONAL AND PSYCHOLOGICAL
C  MEASUREMENT, 1972, 32, 811-814.
C
C  PROBLEM CARD 1
C
C  COLUMNS 1-4    NUMBER OF DATA SETS TO BE READ (I.E., THE NUMBER OF
C                 TIMES PROBLEM CARDS 2-4WILL BE REPEATED).
C                 (I4 FORMAT)
C
C  PROBLEM CARD 2
C
C  COLUMNS 1-3    ALPHA, LEVEL OF SIGNIFICANCE FOR THE F TEST.
C                 (F3.2 FORMAT)
C          4-6    NUMBER OF PHI'S TO BE READ.  (I3 FORMAT)
C          7-9    NUMBER OF N'S TO BE READ.  (I3 FORMAT)
C        10-43    PHI (SUB 0), HYPOTHESIS VALUE.  (F4.2 FORMAT)
C
C  PROBLEM CARD 3
C
C  COLUMNS 1-3    FIRST VALUE OF PHI.
C          4-6    SECOND VALUE OF PHI, ETC., UP TO 20 VALUES.
C                 (20F3.1 FORMAT)
C
C  PROBLEM CARD 4
C
C  COLUMNS 1-4    FIRST VALUE OF N.
C          5-8    SECOND VALUE OF N, ETC., UP TO 20 VALUES.
C                 (20F4.0 FORMAT)
C*************************************************************************
C*************************************************************************
      DIMENSION PHI(20),TOTNUM(50)
      DOUBLE PRECISION D1,D2,ALPHA,F,FPOW,P
      READ 11, NJET
11    FORMAT(I4)
      DO 10 JET = 1,NJET
      READ 7,ALPHA,NPHIS,NTOT,PHO
7     FORMAT(F3.2,2I3,F4.2)
      READ 1,(PHI(J),J=1,NPHIS)
1     FORMAT(20F3.1)
      READ 6,(TOTNUM(J),J=1,NTOT)
6     FORMAT(20F4.0
      IACC = 5
      PRINT 8,NTOT,NPHIS,ALPHA
8     FORMAT(1H1' /////////////////////////////////////////////////////'//'
     C THE ALPHA LEVEL FOR THE NEXT '14' VALUES OF N'//'  WITH '14'  VA
     CLUES OF PHI, IS 'F5.2//'  /////////////////////////////////////////
     C///////////////////')
```

```
      DO 4 I1 = 1,NTOT                                              RE-P0053
      NUMTOT=TOTNUM(I1)                                             RE-P0054
      PRINT 9,NUMTOT,PHO                                            RE-P0055
9     FORMAT(1HO' THE TOTAL NUMBER OF OBSERVATIONS IS 'I6' THE NULL HYPORE-P0056
     CTHESIS HAS A VALUE AT 'F5.2)                                  RE-P0057
      JEND=TOTNUM(I1)/2.0                                           RE-P0058
      DO 4 I2 = 1,NPHIS                                             RE-P0059
      PRINT 5, PHI(I2)                                              RE-P0060
5     FORMAT(1HO'*************************************************'//' RE-P0061
     C THE PHI COEFFECIENT IS 'F6.3,//' *****************************RE-P0062
     C*******************')                                         RE-P0063
      DO 2 J=2,JEND                                                 RE-P0064
      RJ = J                                                        RE-P0065
      JNUM=TOTNUM(I1)/RJ                                            RE-P0066
      D1=J-1                                                        RE-P0067
      D2 = JNUM * J - J                                             RE-P0068
      CALL FSTAT(D1,D2,ALPHA,F,IACC)                                RE-P0069
      RJNUM = JNUM                                                  RE-P0070
      FPOW = F 8((1.+ RJNUM * PHO)/(1. + RJNUM * PHI(I2)))          RE-P0071
      CALL PLEVEL(D1,D2,FPOW,P)                                     RE-P0072
      IF(P.GE..9995D0) GO TO 12                                     RE-P0073
      PRINT 3, J,JNUM,FPOW,P                                        RE-P0074
3     FORMAT (1HO, ' NO. OF RANDOM LEVELS ', I5,' NO. OF PEOPLE IN A LEVRE-P0075
     *EL ',I5,//' F VALUE TO CALCULATE POWER ', F8.3, ' POWER VALUE',F10RE-P0076
     *.5)                                                           RE-P0077
      GO TO 2                                                       RE-P0078
12    PRINT 3, J,JNUM,FPOW,P                                        RE-P0079
      GO TO 4                                                       RE-P0080
2     CONTINUE                                                      RE-P0081
4     CONTINUE                                                      RE-P0082
10    CONTINUE                                                      RE-P0083
      STOP                                                          RE-P0084
      END                                                           RE-P0085
```

Appendix B[1,2]

[1]Only the MAINLINE of the program is included here. The subroutines FSTAT and PLEVEL are very long and were excluded. FSTAT returns the F statistic (F) given alpha (ALPHA) degrees of freedom (D1, D2); PLEVEL returns the probability of a given F statistic (P) given degrees of freedom (D1, D2). Similar routines should be available at most computer centers.

[2]This program requires only one data card which contains:

Columns	Variable	Format
1-3	Alpha level	F3.2
4-7	θ	F4.2
8-11	θ_0	F4.2
12-15	RN, Sample size	F4.0

Two-Way Random-Effects
Analysis of Variance

```
C  THIS PROGRAM FINDS POWER VALUES FOR A TWO-WAY RANDOM-EFFECTS
C  ANALYSIS OF VARIANCE.
C
C
      DOUBLE PRECISION D1,D2,ALPHA,F,FPOW,P
      READ 1,ALPHA,PHI,PHO,RN
    1 FORMAT(F3.2,2F4.2,F4.0)
      PRINT 2,ALPHA,PHI,PHO,RN
    2 FORMAT('1THE VALUE OF ALPHA IS'F5.2//' THE VALUE OF PHI IS'F6.2//'
     COTHE VALUE OF PHI(SUBO), THE HYPOTHEISI VALUE IS'F6.2//'OTHE NUMBE
     CR OF OBSERVATIONS IS'F6.0/////)
      N = RN
      KEND = RN/4.0
      IACC = 5
      DO 3 LLA = 1,3
      GO TO (20,21,22),LLA
   20 PRINT 6
    6 FORMAT ('1THE FOLLOWING ARE POWER VALUES FOR THE INTERACTION'////)
      GO TO 16
   21 PRINT 7
    7 FORMAT ('1THE FOLLOWING ARE POWER VALUES FOR FACTOR A'//' FACTOR A
     1 HAS I LEVELS'////)
      GO TO 16
   22 PRINT 8
    8 FORMAT('1THE FOLLOWING ARE POWER VALUES FOR FACTOR B'//
     1' FACTOR B HAS J LEVELS'////)
   16 PRINT 1111
 1111 FORMAT (7X,'K, THE NO.'T29,'J,THE NO. OF',T49,'THE NO. OF'
     CT69,'NUMERATOR',T86,'DENOMINATOR'T104,'F',T122,'POWER')
      PRINT 1112
 1112 FORMAT (7X,'OF OBSERVATIONS',T29,'LEVELS IN',T49,'LEVELS IN'
     CT69,'DEGREES OF',T86,'DEGREES OF',T104,'STATISTIC')
      PRINT 1113
 1113 FORMAT (7X,'IN A CELL',T29,'FACTOR B',T49'FACTOR A'
     CT69,'FREEDOM',T86,'FREEDOM')
      DO 3 K = 2,KEND
      RK=K
      JEND = N/(K*2)
      DO 3 J=2,JEND
      I = N/(K*J)
      GO TO (10,11,12),LLA
   10 D1 = (I-1 * (J-1)
      D2 = I*J*(K-1)
      CALL FSTAT(D1,D2,ALPHA,F,IACC)
      FPOW = F * ((1. +RK * PHO)/(1. + RK*PHI))
      GO TO 5
   11 D1 = I-1
      D2 = (I-1 * (J-1)
      RJ = J
      CALL FSTAT(D1,D2,ALPHA,F,IACC)
```

```
      FPOW = F * ((1. + RJ*RK*PHO)/(1. + RJ*RK*PHI))
      GO TO 5
   12 D1 =J-1
      D2 = (I-1) * (J-1)
      RI = I
      CALL FSTAT(D1,D2,ALPHA,F,IACC)
      FPOW = F * ((1. + RI*RK*PHO)/(1. + RI*RK*PHI))
    5 CALL PLEVEL(D1,D2,FPOW,P)
      PRINT 4,K,J,I,D1,D2,F,P
    4 FORMAT (7X,T12,I3,T33,I3,T53,I3,T68,F6.0,T88,F6.0,T103,F8.3,
     CT120,F7.3)
    3 CONTINUE
      STOP
      END
```